

CBCS SCHEME

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17MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2021 Engineering Mathematics – III

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Obtain the Fourier series of $f(x) = x(2\pi - x)$ in $0 \leq x \leq 2\pi$. (08 Marks)
- b. Obtain the Fourier series for the function $f(x) = \begin{cases} 1 + 4\frac{x}{3} & \text{in } -\frac{3}{2} < x \leq 0 \\ 1 - 4\frac{x}{3} & \text{in } 0 \leq x < \frac{3}{2} \end{cases}$ (06 Marks)
- c. Expand $f(x) = 2x - 1$ as a Cosine half range Fourier series in $0 < x < 1$. (06 Marks)

OR

- 2 a. Obtain the constant term and the coefficients of the first Cosine and Sine terms in the Fourier expansion of 'y' from the table (08 Marks)

| | | | | | | |
|---|---|----|----|----|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| y | 9 | 18 | 24 | 28 | 26 | 20 |

(06 Marks)

- b. Obtain the Fourier series of $f(x) = |x|$ in $-\pi \leq x \leq \pi$. (06 Marks)
- c. Show that the sine half range series for the function $f(x) = \ell x - x^2$ in $0 < x < \ell$ is

$$\frac{8\ell^2}{\pi^3} \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \sin\left(\frac{2n+1}{\ell}\right) \pi x. \quad (06 \text{ Marks})$$

Module-2

- 3 a. If $f(x) = \begin{cases} 1 & \text{for } |x| \leq a \\ 0 & \text{for } |x| > a \end{cases}$, find the infinite Fourier transform of $f(x)$ and hence evaluate $\int_0^\infty \frac{\sin x}{x} dx$. (08 Marks)
- b. Find the Fourier Cosine transform of e^{-x} (06 Marks)
- c. Solve by using Z-transforms: $y_{n+2} - 4y_n = 0$, given $y_0 = 0$ and $y_1 = 2$. (06 Marks)

OR

- 4 a. Find the Fourier Sine transform of $\frac{e^{-ax}}{x}$, $a > 0$. (08 Marks)
- b. Find the Z-transform of $\sin(3n + 5)$. (06 Marks)
- c. Find the inverse Z-transform of $\frac{2z^2 + 3z}{(z+2)(z-4)}$. (06 Marks)

Module-3

- 5 a. Find the coefficient of correlation for the data

| | | | | | | | | | | |
|---|---|---|----|---|----|----|----|----|----|----|
| x | 1 | 3 | 4 | 2 | 5 | 8 | 9 | 10 | 13 | 15 |
| y | 8 | 6 | 10 | 8 | 12 | 16 | 16 | 10 | 32 | 32 |

(08 Marks)

- b. Fit a straight line to the following data

| | | | | | |
|-----------------------|------|------|------|------|------|
| Year | 1961 | 1971 | 1981 | 1991 | 2001 |
| Production (in tons) | 8 | 10 | 12 | 10 | 16 |

(06 Marks)

- c. Compute the real root of $x \log_{10}x - 1.2 = 0$ by Regula – Falsi method. Carry out three iterations in (2, 3). (06 Marks)

OR

- 6 a. Obtain the lines of Regression for the following values of x and y

| | | | | | |
|---|---|---|---|---|---|
| x | 1 | 2 | 3 | 4 | 5 |
| y | 2 | 5 | 3 | 8 | 7 |

(08 Marks)

- b. Fit an exponential curve of the form $y = ae^{bx}$ for the following data

| | | | | | | |
|----------------|-----|----|----|---|---|----|
| No. of petals | 5 | 6 | 7 | 8 | 9 | 10 |
| No. of flowers | 133 | 55 | 23 | 7 | 2 | 2 |

(06 Marks)

- c. Find a real root of $x \sin x + \cos x = 0$ near $x = \pi$. Correct to four decimal places, using Newton – Raphson method. (06 Marks)

Module-4

- 7 a. Given $\sin 45^\circ = 0.7071$, $\sin 50^\circ = 0.7660$, $\sin 55^\circ = 0.8192$, $\sin 60^\circ = 0.8660$, find $\sin 57^\circ$ using an appropriate interpolation formula. (08 Marks)

- b. Use Newton's divided difference formula to find $f(4)$ given the data

| | | | | |
|------|----|---|----|-----|
| x | 0 | 2 | 3 | 6 |
| f(x) | -4 | 2 | 14 | 158 |

(06 Marks)

- c. Using Simpson's 1/3rd rule, evaluate $\int_0^{\pi/2} \sqrt{\cos \theta} d\theta$ by dividing $[0, \pi/2]$ into 6 equal parts. (06 Marks)

OR

- 8 a. From the following table find the number of students who have obtained less than 45 marks

| | | | | | |
|-----------------|-------|-------|-------|-------|-------|
| Marks | 30-40 | 40-50 | 50-60 | 60-70 | 70-80 |
| No. of Students | 31 | 42 | 51 | 35 | 31 |

(08 Marks)

- b. Using Lagrange's interpolation formula fit a polynomial of the form $x = f(y)$

| | | | |
|---|---|----|----|
| x | 2 | 10 | 17 |
| y | 1 | 3 | 4 |

(06 Marks)

- c. Evaluate $\int_0^1 \frac{x}{1+x^2} dx$ by Weddle's rule taking seven ordinates. (06 Marks)

Module-5

- 9 a. Verify Green's theorem in a plane for $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where 'C' is the boundary of the region enclosed by $y = \sqrt{x}$ and $y = x^2$. (08 Marks)
- b. Verify Stoke's theorem for $\vec{F} = (x^2 + y^2)i - 2xyj$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. (06 Marks)
- c. Derive Euler's equation $\frac{\partial t}{\partial y} - \frac{d}{dx} \left[\frac{\partial t}{\partial y^1} \right] = 0$. (06 Marks)

OR

- 10 a. Use Gauss divergence theorem to evaluate $\iint_S \vec{F} \cdot \hat{n} ds$ over the entire surface of the region above xy plane bounded by the cone $z^2 = x^2 + y^2$ the plane $z = 4$ where $\vec{F} = 4xz i + xyz^2 j + 3z K$. (08 Marks)
- b. Prove that geodesics of a plane are straight lines. (06 Marks)
- c. Find the extremal of the functional $\int_0^{\pi/2} (y^2 - y^{1^2} - 2y \sin x) dx$ under the end conditions $y(0) = y(\pi/2) = 0$. (06 Marks)